Theorems and friends[1]

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Contents

Abstract

LaTeX offers an incredibly large set of facilities for typestting mathematically-rich texts. This article examines one such feature: handling theorems in mathematical texts.

1 Theorems and friends

Theorems are an integral part of mathematics. In fact, there are several shades of "theorems" (listed below) [3].

- theorem
- axiom/postulate
- conjecture/hypothesis
- proposition

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- lemma
- corollary
- converse
- generalisation
- user-defined

LaTeX keeps a separate counter for each of the above, and manages them sutomatically as you add/delete any specific item. They can also be used for cross-referencing purposes.

A **theorem** is a statement that has been proven on the basis of previously established statements, such as other theorems, and generally accepted statements, such as axioms

An **axiom / postulate** is a statement that is accepted without proof and regarded as fundamental to a subject. Historically these have been regarded as "self-evident", but more recently they are considered assumptions that characterize the subject of study. In classical geometry, axioms are general statements, while postulates are statements about geometrical objects. A large collection of famous axioms is available in [4]

A **definition** is also a statement which is accepted without proof, since it simply gives the meaning of a word or phrase in terms of known concepts.

A **conjecture / hypothesis** is an unproved statement that is believed true. To be considered a conjecture, a statement must usually be proposed publicly, at which point the name of the proponent may be attached to the conjecture, as with Goldbach's conjecture. Other famous conjectures include the Collatz conjecture and the Riemann hypothesis. On the other hand, Fermat's Last Theorem has always been known by that name, even before it was proved; it was never known as "Fermat's conjecture" (probably because of the enigmatic claim Fermat made, before he died).

A **proposition** is a theorem of lesser importance. This term sometimes connotes a statement with a simple proof, while the term theorem is usually reserved for the most important results or those with long or difficult proofs. Some authors never use "proposition", while some others use "theorem" only for fundamental results. In classical geometry, this term was used differently: In Euclid's Elements (c. 300 BCE), all theorems and geometric constructions were called "propositions" regardless of their importance.

A **lemma** is a "helping theorem", a proposition with little applicability except that it forms part of the proof of a larger theorem. In some cases, as the relative importance of different theorems becomes more clear, what was once considered a lemma is now considered a theorem, though the word "lemma" remains in the name. Examples include Gauss's lemma, Zorn's lemma, and the fundamental lemma.

A **corollary** is a proposition that follows with little proof from another theorem or definition.[7] Also a corollary can be a theorem restated for a more restricted special case. For example, the theorem that all angles in a rectangle are right angles has as corollary that all angles in a square (a special case of a rectangle) are right angles.

The **converse of a theorem** is a statement formed by interchanging what is given in a theorem and what is to be proved. For example, the isosceles triangle theorem states that if two sides of a triangle are equal then two angles are equal. In the converse, the premise (that two sides are equal) and what is to be proved (that two angles are equal) are swapped. So, the converse is the statement that if two angles of a triangle are equal then two sides are equal. In this example, the converse can be proved as another theorem, but this is often not necessary.

A **generalization** is a theorem which includes a previously proved theorem as a special case and hence as a corollary.

User-defined category : In addition to the above, LaTeX gives you a whole lot of possibilities to define your own categories of theorems. You can make – theorems, lemmas, corollaries, conjectures, example, remark, definitions, hunches etc. , just about anything. You can use a wide choice of styles in which these "theorems" will be visually presented. Use `\newtheorem` and `\theoremstyle` in the Preamble, to create your own categories of theorems, lemmas, hunches etc. LaTeX keeps a separate counter for each of these categories, so you can use them in cross-referencing.

2 User-defined categories

We present two user-defined categories which are treated just like theorems, by LaTeX :

- wild hunches
- non-theorems

We included these two categories in the preamble part of this document, using

```
\theoremstyle{break} \newtheorem{hunch}{The law of wild-hunches}
\theoremstyle{plain} \newtheorem{ahunch}{Non-theorem}
```

Here is what we can do.

The law of wild-hunches 1
All hunches do not become theorems. But, all theorems started out as hunches.

Non-theorem 1. *Sometimes, theorems turn out to be fakes, and then, they become plain hunches.*

The law of wild-hunches 2
This is a dummy wild-hunch 2, just to show you how theorem numbers get incremented automatically.

Non-theorem 2. *This is a dummy nontheorem 2. All hunches are not theorems, and all theorems are not just hunches.*

And now, we see two theorems, in the classical sense of the term:

Theorem 1. *Pythagoras Theorem*

This is the celebrated theorem ascribed to the great **Pythagoras** by his followers: *The square of the length of the hypotenuse (c) of a right angled triangle equals the sum of the squares of the lengths of the other two sides (a,b).* In other words,:

$$c^2 = a^2 + b^2 \tag{1}$$

Theorem 2. *Fermat's Last Theorem*

Poor **Louis Fermat** was not so lucky. He could'nt go beyond three in his famous equation.

Thus spake Louis Fermat : *There are no integral solutions for the equation :*

$$x^n + y^n = z^n for\ n\ \geq 3 \quad (2)$$

and died, before he could publish the proof of his claim.

Moral of the story: Pythagoras made a positive statement about something. There are a hundred ways to prove Pythagoras right. Fermat had a negative statement to make. Fermat had to wait 200 years to prove his theorem. Is this proof of the power of positive thinking ?

3 Closing remarks

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Citation details of this article are given in [1].

References

[1] S. Parthasarathy, Theorems and corollaries,
`https://drpartha.org.in/publications/theoremcorollaries.pdf`

[2] S. Parthasarathy, When grace meets beauty,
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[3] Wikipedia, https://en.wikipedia.org/wiki/Theorem

[4] Wikiwand, List of axioms,
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About the author

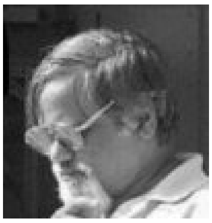

Fig. 1: Dr. Partha

S. Parthasarathy (aka Dr. Partha) brings with him, a very rich experience (since 1980) in the software industry, both as an employee of a large company, and as an independent entrepreneur. He holds an Enginering Doctorate (Docteur Ingenieur) from Grenoble, France, and was also a Post-Doctoral Scientist in Paris, France. He has visited and taught at several institutions worldwide (France, China, Thailand, Indonesia, Germany, Spain, UK, Poland, Nepal, India). He is the author of several research publications, popular articles, and educational CDROMs. He is an Editor of the international journal "Engineering Applications of Artificial Intelligence" , published from the UK by Pergamon/Elsevier Press. He is an aggressive supporter of the Free and Open Source Software (FOSS) movement, and is a regular contributor to the international effort on Linux. His contributions are part of all major Linux distributions worldwide. One of his contributions has been translated (from English) into seven different languages !

Parthasarathy speaks French and English (and many other languages) fluently. He is a Senior Member of IEEE and a Fellow of IETE.

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