Radians of a circle

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" Pi is God's telephone number. "
– Alexander Graham Bell

1 The notorious π

Mathematicians seem to be a crazy tribe which prefers weird notations when there are easier and good-looking notations available. The case of units for angles is an excellent example. Or, is it ?

We saw in an earlier article why dividing a circle into 360 degrees was a bad idea[1], born out of an astronomical blunder and ignorance. We need a less defective (in origin) measure of angles. Enter **radian**, a unit for angles, defined by the international body SI.

The radian (SI symbol rad) is the SI unit for measuring angles, and is the standard unit of angular measure used in many areas of mathematics.

By definition, the length of an arc of a unit circle is numerically equal to the measurement in radians of the angle that it subtends. Since the circumference C of a circle of radius r is given by: $C = 2\pi r$, the complete circle subtends

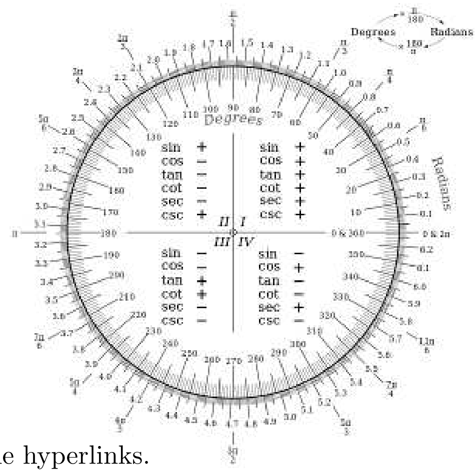

Figure 1: Convert degrees to radians

*All texts shown in winered color are clickable hyperlinks.
[1]https://bit.ly/3ytq5PH

an angle of 2π radians. Thus a semicircle subtends π radians (which corresponds to 180°). Since π radians = 180° , we get the simple relation between radians and degrees as 180° / π = 57.295779513° .

We can thus move from degrees to radians (and back) by a simple arithmetic operation. However, in daily use, it is still more convenient to divide a circle into degrees. It is always easier to handle whole numbers (integers) rather than clumsy fractional numbers. The debate is similar to the choice between using metric units of length (metres) and imperial units (feet, inches). We still talk of milestones and not kilometrestones ! There is however, a (in fact, many) very good mathematical reason(s) why radians are used so commonly in mathematics and physics.

Radians are the most useful angular measure in calculus (and trigonometry), since radians have a mathematical "naturalness" that leads to a more elegant formulation of a number of important results. Most notably, results in analysis involving trigonometric functions are simple and elegant when the functions' arguments are expressed in radians.

They allow derivative and integral identities to be written in simple terms, For example,

$$\frac{d}{dx}\, sin(x) = cos(x)$$

for x measured in radians. With angles as radians, computation of areas, volumes and limits become straight forward.

The trigonometric functions also have simple and elegant series expansions when radians are used. For example, the following is the Taylor series for sin x (x in radians) :

$$\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots .$$

If x were expressed in degrees, then the series would contain messy factors involving powers of $\pi/180$.

The following equation (Euler's identity) , considered to be the most lovely equation in mathematics :

$$e^{i\pi} + 1 = 0$$

would become a horrible-looking mess, if we use degrees in the exponent.

2 How big is π ?

If π radians make 180° , then how big is π exactly (expressed in decimal digits) ?

The history of calculating the value of π is long. The decimal approach to pi is an endless rabbit hole, but people have followed it right down to the present day. Now, of course, computers are used, and gazillions of decimal places have been computed. For a really long time, math wizards have been on a quest to figure out pi. Back in ancient times, folks like the Egyptians and Babylonians needed a decent pi for practical math stuff. Around 250 BC, a Greek math whiz named Archimedes came up with a cool way to get pi super accurately. Jump to the 5th century AD, and Chinese math buffs got Ï right to seven digits, while Indian math brains hit five digits, both using cool geometric tricks. It took about a thousand years more for someone to discover a nifty formula for Ï based on a never-ending series. In more recent times, the Indian genius Raamanujan had an innovative and interesting expression for $1/pi$ as an infinite sum .

The symbol π itself was introduced by William Jones in 1706, but not used universally until the late 19th century.

The first, and worst, estimate is π= 3, sometimes attributed to the Bible. This value comes from Kings 7:23, which mentions a "molten sea" made by King Solomon, "ten cubits from brim to brim" such that "thirty cubits would encircle it completely." If we assume that the object being encircled is indeed circular, this implies that $\pi = 3$.

Some school teachers make the classic blunder of writing (and teaching) $\pi = 22/7$, whereas they should be strictly using $\pi \approx 22/7$. They forget the fact thatπ is irrational. Writing it as p/q (where p, q are integers) would be a crime.They are usually ignorant of the fact there is a much better approximation to π. The value 355/113, due to the Chinese mathematician Zu Chongzhi in the 5th century CE, gives the value of π correct to six decimal places. Like Archimedes, Zu approximated the circle by polygons and knew that he had not found the exact value of π. The idea of approximating the circle by a polygon with a large number of sides can be pushed as far as one has patience to carry out the calculations involved. For a long time the record holder was Ludolph van Ceulen, an otherwise obscure 16th-century Dutch mathematician who calculated the first 35 decimal places of π by this method. Van Ceulen was remembered by the term "Ludolph's number" for π, long after his record had been surpassed.

It is, in any case, futile to write π in decimal representation, since π is "transcendental". The fractional part, written in decimal can go on forever. It is like trying to hold a million-legged octopus. π can be used as such, without ever having to express it as an approximation of the form p/q (remember π is irrational), or expressing it as a number with decimal digits (remember π

is a transcendental number).

The downside of radians: Radians are admittedly clumsier than degrees. It is not easy to subdivide a radian into smaller, integer parts (like 1/3, 1/,4, 1/5, 1/6 etc.). Remember, π is an irrational number.

Moral of the story : Degrees are for schoolchildren, radians are for adults.

Closing remarks : Constructive comments, remarks and suggestions are always welcome.

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