Paradoxes

S. Parthasarathy
drpartha@gmail.com

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Abstract

A paradox is a fascinating and intriguing concept, studied by linguists, logicians, and philosophers.

This article presents an overview of paradoxes and presents some examples

1 What are paradoxes ?

Simply stated, a paradox (or a logical paradox) is a statement that is apparently true, but contradicts itself (might be true and false at the same time, or can neither be true nor false). A very interesting resource for such discussions is the Internet Encylcopedia of Philosophy [2]

1.1 A simple example

The most common example of a paradox is the statement : *This statement is false*. This is often called "the Liar Sentence".

Let L be the Classical Liar Sentence. If L is true, then L is false. But we can also establish the converse, as follows. Assume L is false. Because the Liar Sentence is just the sentence that says L is false, the Liar Sentence is therefore true, so L is true. We have now shown that L is true if, and only if, it is false. Since L must be one or the other, it is both. [2]

A variant of the above statement is when a person states *I am a liar*. An interesting extension of the above is given in [2]:

- The following sentence is true.
- The following sentence is true.
- The following sentence is true.
- The first sentence in this list is false.

Imagine entering a village where some men are liars, and some are honest (speak the truth always). How do you recognise a liar/honest person, just by asking that person ? The puzzle becomes much more confusing, if the men chose to be liars/honest men, on randomly chosen days. The Goodman Principle of induction is based on this scenario.

1.2 Have you stopped beating your wife ?

A paradoxical scenario can be extended to ridiculous every-day situations. An example of this would be when you are asked, "Have you stopped beating your wife?" This is not a simple question which can be answered in YES or NO, because the answer may lead to some undesired misinterpretation.

YES : I used to beat her, now I don't.

NO : I used to beat her. I continue to beat her.

Your audience might still need to be taken slowly through the answer, before they clearly see the point you are trying to make..

1.3 Smullyan's trap

Consider this puzzle which Raymond Smullyan [1] used for getting his girl friend to marry him. Raymond Smullyan [1] is an exceptionally talented person. He is an amazing mathematician, logician and magician, concert pianist, and an author, all rolled into one. The following story was narrated by Smullyan himself in a video presentation:

When Smullyan, the master of puzzles [4], met his date, he challenged her to a logical puzzle.

The contract : Smullyan was to make a statement.

- If the statement were true, the date had to give Smullyan an autograph.
- If the statement were not true (i.e. it is false), the date should not give Smullyan the autograph

The innocent date did not see a trap, and agreed to the contract.

Now, the statement Smullyan made was:

You will give me neither an autograph nor a kiss

If this is a true statement, the date would give Smullyan neither an autograph nor a kiss. This would contradict the contract made earlier (a true statement must get an autograph). On the other hand, if the statement was not true, it had to be false. In which case, the date would give either a kiss or an autograph. By contract, the date could not give her autograph for a false statement, and had to give Smullyan a kiss. Smullyan built up a "double or quit" game based on this puzzle and collected all the kisses he needed, till the point where the date had to quit by agreeing to marry Smullyan.

This is one clinching reason where paradoxes could be beneficial (to some) and harmful (to others). Hence it is important to study and understand logical paradoxes.

1.4 Paradoxical rules

Consider the statement *Every rule has an exception.* If this statement is true, it has an exception therefore the statement is not true. And, if this statement is false, the rule has no exception. Hence it is true.. That makes it an exceptionally paradoxical paradox !

1.5 Mathematical paradoxes

Sometimes, you need to use maths and symbolic logic to recognise and break a paradox. Here is an example.

1.5.1 The two envelopes problem

This problem is also known as Monty Hall problem [3]

Two Envelopes Paradox: You are taking part in a game show. The host offers you two envelopes, each containing some money. She tells you that one envelope contains exactly twice as much as the other, but does not tell you which is which. You may choose one, keeping the money it contains.

Since you have no way of knowing which envelope contains the larger sum, you pick one at random. The host asks you to open the envelope. You do so and take out a check for $40,000.

The host now says that you have a chance to change your mind and choose the other envelope. If you don't know anything about probability theory, particularly expectations, you probably say to yourself, the odds are fifty-fifty that you have chosen the larger sum, so you may as well stick with your first choice.

On the other hand, if you know a bit (though not too much) about probability theory, you may well try to compute the expected gain due to swapping. The chances are you would argue as follows. The other envelope contains either $20,000 or $80,000, each with probability .5. Hence the expected gain of swapping is

[0.5 x 20,000] + [0.5 x 80,000] - 40,000 = 10,000

That's an expected gain of $10,000. So you swap.

But wait a minute. There's nothing special about the actual monetary amounts here, provided one envelope contains twice as much as the other. Suppose you opened one envelope and found $M. Then you would calculate your expected gain from swapping to be
[0.5 x M/2] + [0.5 x 2M] - M = M/4

and since M/4 is greater than zero you would swap.

Okay, let's take this line of reasoning a bit further. If it doesn't matter what M is, then you don't actually need to open the envelope at all. Whatever is in the envelope you would choose to swap.

Well, if you don't open the envelope, then you might as well choose the other envelope in the first place. And having swapped envelopes, you can repeat the same calculation again and again, swapping envelopes back and forth ad-infinitum. There is no limit to the cumulative expected gain you can obtain. But this is obviously absurd.

And there's the paradox. Now, you will need some mathematics (probability theory) to break the paradox. Is there something wrong with the computation of the expected gain from swapping? This analysis is reserved for a sequel to this article.

2 Closing remarks

The author invites suggestions, queries and remarks from readers of this article. The author invites particularly members of the academic community (both teachers and students) to react.

References

[1] Raymod Smullyan, https://en.wikipedia.org/wiki/RaymondSmullyan

[2] Internet Encylcopedia of Philisophy, http://www.iep.utm.edu/par-log/

[3] MontyHall problem , https://en.wikipedia.org/wiki/Monty_Hall_problem

[4] Raymond Smullyan, *The lady or the tiger ? and other logical puzzles*, Times Books, New York, 1982.

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