A special family of numbers [1] [2]

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1 A very special family of numbers

We will start this article with a maths based pun – *all prime numbers are odd !* Let us now explore the truth in this statement. Of course, all prime numbers are odd numbers, but there is nothing odd about a prime number (pun intended). A "prime number" is defined in Section #2

In this article, the term "number" would mean natural numbers.

There are numbers and numbers. Mathematicians like to study their properties collectively instead of grappling with one number at a time. To do this, they group numbers into "families". The members which exhibit a set of common properties, are put into a family
(see https://drpartha.org.in/publications/mathnumbers.pdf).

Some numbers happen to be fundamental for understanding other numbers. These are primary numbers, or "prime numbers", in short. To take an analogy, any colour can be seen as a combination of the three "primary colours" – red, green, blue. Mixing them in the right proportion can yield any desired colour.

The dictionary meaning of "prime" may be stated as :: main or most important. It is used as an adjective, to indicate first, important or best, like "prima donna" in a music concert. Similarly, prime numbers form an important family of numbers. Here are a few examples of the usage of *prime*

[1]This pdf document was created using LaTeX . Texts marked in wine-red colour are click-sensitive hyperlinks.

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- The president is a *prime* target for the assassin's bullet.
- The *Prime* Minister will inaugurate the new railway line.
- The hotel is in a *prime* location in the city.

2 What are prime numbers ?

Some numbers are special, because they have an important property. They are justifiably called *prime* numbers.

A natural number can be either a prime number or not be a prime number (aka composite number). In a mathematical context, a prime number (or a prime) is a natural number greater than 1 that is not a product of two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. The divisor of composite numbers are called its factors. By commonsense, since composite numbers are products of its factors, they have to be greater than 1.

By extension, Prime numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder. This property could appear to be a handicap, but prime numbers have extremely important uses in mathematics and in many other fields. Number theory has been a fertile area of research in the past two centuries. Prime numbers are the basis of some profound analysis and results in number theory, and have given rise to several breakthroughs in fields like cryptography. Prime numbers can often be seen in various other contexts as well.

If you look carefully, prime numbers pop up from almost everywhere. Take the number of players in your football team – 13 ! Or, the number of players in a cricket team – 11, Or, the number of players in a kabbadi team (a popular sport in India) – 7, Or, the number of players in a chess tournament – 2. Or, take the number of days in a week – 7. Or, take the number of days in a year – 365, which can be written as 73*5 (which are both primes).The list can go on endlessly. The most common example (also my favourite) is the 31 in Baskin Robbins ice cream ! Take a good look at their logo. The 31 is cleverly hidden in their logo. It is hidden, but it is visible to the naked eye. This is symbolic of the mystery of prime numbers. Prime numbers are hiding in all natural numbers, as explained in the two theorems mentioned below.

Two important theorems justify the use of the qualificative “prime” for prime numbers. They can be seen as members of the *family of prime numbers*

1. Fundamental theorem of arithmetic

2. Goldbach’s conjecture

3 Fundamental theorem of arithmetic FTA

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that *every natural number greater than 1 either is a prime itself or is the product of a unique combination of prime numbers* (FTA).

4 Goldbach’s conjecture GC

The German mathematician Christian Goldbach proposed that *Every natural number greater than 2 is either odd or the sum of two prime numbers* (GC). This statement has not yet been formally proved, or disproved, and is hence still considered a conjecture. Goldbach’s conjecture (GC) is one of the oldest and best-known unsolved problems in number theory and all of mathematics.

5 Endless discoveries

Notice that a natural number could be a prime (prime number) or not (composite number). Two cases arise:

- If it is not a prime (a composite), it can be expressed as a product of primes (FTA).

- If it is a prime, it can be expressed as a sum of two primes (GC).

Whichever way you see it. No matter which integer number your choose, you have prime numbers hiding inside all of them. This is just like the primary colours hiding inside whatever colour you see.

In short, the ubiquity of prime numbers can be visualised in the following fictitious scenario. Imagine the mythical deluge happening again. Imagine all prime numbers vanish from this earth. By the above theorems, we can say goodbye to all numbers, and to the whole of arithmetic (and mathematics) !

Prime numbers can be an extremely fascinating and profound subject of study and discoveries. It is a major contributor to number theory and has applications in fields like algebraic number theory, cryptography. One common activity would be to estimate the distribution of primes in a series of consecutive integers. Enumerating all properties of prime numbers is still a formidable task. Such properties can be used to derive important relationships in number theory, and eventually to interesting applications like cryptography. The practical problem of finding/proving whether a given number is prime or not (primality testing) has been a challenge for many mathematicians and number theorists. Primality testing is an important part of the search for prime numbers. A world-wide search for prime numbers, which needs primality testing, involving collaborative computing GIMPS (https://www.mersenne.org/) has yielded some dramatically large prime numbers.

6 Closing remarks

Readers are requested to send all remarks, criticisms and suggestions to the author at drpartha@gmail.com.

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