# Logic of reasoning and proof

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This tutorial is a sequel to another related tutorial [1].

In mathematics, a statement is not accepted as valid or correct unless it is accompanied by a proof. This insistence on proof is one of the things that sets mathematics apart from other subjects.

A **proof** is an argument from **premises** to a **conclusion**. The conclusion is the statement that you wish to prove.

Example:

*Premise* – A,B,C are the angles of a triangle.
*Conclusion* – $A + B + C = 180\ ^\circ$

The couple [Premise,Conclusion], when proved, becomes a theorem [3]. Well-defined theorems and axioms are the stepping stones of all proofs [3] [4] .

Each step of the proof follows the laws of logic (in case of logical proofs), or other laws of mathematics (in case of mathematical proofs) . In the rest of this article, our focus is on logical proofs as described in [2]. The idea is to operate on the premises using rules of inference until you arrive at the conclusion. Like most proofs, logic proofs usually begin with premises — statements that you're allowed to assume. Rules/axioms of logic are repeatedly applied to the premises, till we arrive at the desired conclusion.

By comparison, in the case of mathematical proofs (e.g. geometrical proofs), the premises are successively refined using appropriate mathematical theorems and axioms.

Two strategies are commonly involved in this process:

- deductive reasoning
- inductive reasoning

**Deductive reasoning** moves from generalities to specific conclusions. It allows you to take information from two or more statements and draw a logically sound conclusion. Perhaps the biggest stipulation is that the statements upon which the conclusion is drawn need to be true.

Deductive reasoning is a type of deduction used in science and in life. It is when you take two true statements, or premises, to form a conclusion. For example, A is equal to B. B is also equal to C. Given those two statements, you can conclude A is equal to C using deductive reasoning (using an axiom of logic [1]).

This is a very common text-book example of deductive reasoning (also known as a logical syllogism) :

- All men are mortal
- Socrates was a man

Conclusion : Socrates was mortal (of course, that is true !)

Now, let's look at a real-life example.

- All dolphins are mammals.
- All mammals have kidneys.

Using deductive reasoning, you can conclude that all dolphins have kidneys. Remember, for this to work, both statements must be true. Okay, now that you have a good grasp on it, try one more example.

When it comes to deductive reasoning, you can overgeneralize. In these cases, even with two solid and true premises, deductive reasoning goes wrong. Here are a few examples of just that:

All swans are white. Jane is white. Therefore, Jane is a swan.

All farmers like burgers. Jethro likes chicken wings. Therefore, Jethro is not a farmer.

**Inductive reasoning** is akin to deductive reasoning. Inductive reasoning is "bottom up," meaning that it takes specific information and makes a broad generalization that is considered probable, allowing for the fact that the conclusion may not be accurate. The main difference is that, with inductive reasoning, the premises provide some evidence for the validity of the conclusion, but not all.

Inductive Reasoning: The first lipstick I pulled from my bag is red. The second lipstick I pulled from my bag is red. Therefore, all the lipsticks in my bag are red.
Deductive Reasoning: The first lipstick I pulled from my bag is red. All lipsticks in my bag are red. Therefore, the second lipstick I pull from my bag will be red, too.

A more mathematically oriented example would be as follows:

Question: What would be the next number in the series – 1, 3, 5, 7, .... ? If we observe that all the numbers are odd, the answer could be 9, but may not always be the right one. If we assume that all numbers have to be prime, we would choose 11 to be the answer.

This type of reasoning usually involves a rule being established based on a series of repeated experiences.

Premises: An umbrella prevents you from getting wet in the rain. Ashley took her umbrella, and she did not get wet.

Conclusion: In this case, you could use inductive reasoning to offer an opinion that it was probably raining.

Explanation: Your conclusion, however, would not necessarily be accurate because Ashley would have remained dry whether it rained and she had an umbrella, or it didn't rain at all.

Premises: Every three-year-old you see at the park, spends most of his/her time crying and screaming.

Conclusion: All three-year-olds in the park, must spend their time screaming or crying.

Explanation: This would not necessarily be correct, because you havent seen every three-year-old in the world during the afternoon to verify it.

# Closing remarks

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