# The axioms of logic

**S. Parthasarathy**
drpartha@gmail.com

Ref. : logic-axioms.tex
Ver.: 20220307a

**Logic** is the study of correct reasoning or good arguments. Here, logic is considered in the sense of Boolean logic, as contrasted with other logics like fuzzy logic or modal logic. Logic is often defined in a more narrow sense as the science of deductively deriving valid inferences or logical truths. Like all formal systems, logic is also based on some fundamental axioms viz. the law of identity, the law of the excluded middle, and the law of contradiction.

If A is a logical statement (or proposition), all operations involving A obey the following axioms:

- **law of identity** says something like P=P, in other words a thing (or term or statement) is what it is, it is identical to itself.

  The law of identity (P is P ) says that if a statement P , such as It is raining is true, then it is indeed raining, similarly, if it is raining, P is true. Conversely, if P is not true, it is not raining. More generally, it says that the statement P is the same thing as itself and is different from everything else. Applied to all reality, the law of identity says that everything is itself and not something else. This law is self-evident and is often taken for granted. Everything would be total chaos if the law of identity did not always hold. Blue could mean nonblue, dead could mean alive, white could mean black.

- **law of the excluded middle** is something like a statement (or proposition) is either true or false but not both.

  Symbolically, this would mean :

$A \oplus \neg A$

*The cat is in the bag or the cat is out of the bag.* The "or" here is an "exclusive or".

The law of the excluded middle (Either P or non-P ) says that the statement it is neither raining nor not raining is absurd. P is either true or not true, but not both, there is no other alternative.

- **law of contradiction**, which probably should have been called the law of noncontradiction "not both (A and not A)," in other words, a statement and its contradictory are not both true.

  $\neg(A \wedge \neg A)$ is true.

  *It is not possible that the cat is in the bag and the cat is out of the bag*

  The law of noncontradiction says that a state- ment such as It is raining cannot be both true and false in the same sense. Of course it could be raining in Bombay and not raining in Calcutta, but the noncontradiction principle says that it cannot be raining and not raining at the same time in the same place.

Like in the case of all axioms, the above axioms are taken for granted and need no special proof. All logical reasoning and proof are built on the above axioms.

**Reasoning** is the process of asserting a conslusion, based on proof. A proof is an argument (or a series of arguments) from hypotheses (premises) to a conclusion. Each step of the argument follows the laws of logic. The conclusion is the statement that you need to prove. The idea is to operate on the premises using rules of inference until you arrive at the conclusion. Like most proofs, logic proofs usually begin with premises — statements that you're allowed to assume.

To strategies of reasoning are commonly involved in this process:

* deductive reasoning

* inductive reasoning

**Deductive reasoning** moves from generalities to specific conclusions. It allows you to take information from two or more statements and draw a logically sound conclusion. Perhaps the biggest stipulation is that the statements upon which the conclusion is drawn need to be true.

Deductive reasoning is a type of reasoning used in science and in life. It is when you take two (or more) true statements, or premises, to form a conclusion. For example, A is equal to B. B is also equal to C. Given those two statements, you can conclude A is equal to C using deductive reasoning.

Now, lets look at a real-life example.

All dolphins are mammals.

All mammals have kidneys.

Using deductive reasoning, you can conclude that all dolphins have kidneys. Remember, for this to work, both statements must be true. Okay, now that you have a good grasp on it, try a few examples.

When it comes to deductive reasoning, you can overgeneralize. In these cases, even with two solid and true premises, deductive reasoning goes wrong. Here are a few examples of just that:

All swans are white. Jane is white. Therefore, Jane is a swan.

All farmers like burgers. Jethro likes chicken wings. Therefore, Jethro is not a farmer.

**Inductive reasoning** Inductive reasoning is "bottom up," meaning that it takes specific information and makes a broad generalization that is considered probable, allowing for the fact that the conclusion may not be accurate. The main difference is that, with inductive reasoning, the premises provide some evidence for the validity of the conclusion, but not all.

Inductive Reasoning: The first lipstick I pulled from my bag is red. The second lipstick I pulled from my bag is red. Therefore, all the lipsticks in my bag are red.

Deductive Reasoning: The first lipstick I pulled from my bag is red. All lipsticks in my bag are red. Therefore, the second lipstick I pull from my bag will be red, too.

This type of reasoning usually involves a rule being established based on a series of repeated experiences.

Premises: An umbrella prevents you from getting wet in the rain. Ashley took her umbrella, and she did not get wet.

Conclusion: In this case, you could use inductive reasoning to offer an opinion that it was probably raining.

Explanation: Your conclusion, however, would not necessarily be accurate because Ashley would have remained dry whether it rained and she had an umbrella, or it didn't rain at all.

Premises: Every three-year-old you see at the park each afternoon spends most of their time crying and screaming.

Conclusion: All three-year-olds must spend their afternoon screaming.

Explanation: This would not necessarily be correct, because you havent seen every three-year-old in the world during the afternoon to verify it.

## Closing remarks

This article was created by the author, using LaTeX . The LaTeX source of this article can be obtained by sending a request to : drpartha@gmail.com. Please mention the Ref. No., and the Vers. code mentioned at the top of this article. As always, your constructive suggestions and remarks are always welcome.

***