Why is it so difficult to teach (mathematics) ?

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Ref.: difficult.tex
Ver. code: 20161218c

Abstract

This is a revised version of a paper published originally in Mathematical Education, April-June 1993. The paper was written during the author's sabatical year, spent at the United Nations University, International Institute of Software Technology (UNU/IIST), Macau (P R China).

Here, the author uses his experience in Artificial Intelligence (knowledge acquisition), to throw some light on the difficulties of teaching, in general, and teaching mathematics in particular.

1 Perils of teaching

The aim of this essay is to bring out a few points for reflection and debate, on an intriguing aspect of knowledge transfer. In general, what applies to mathematics, also applies to many other branches of knowledge, which explains the parantheses at the end of the above title line. As a matter of fact, the above deceptively simple question has no answer, or rather, no simple answer. So, we do not claim to answer the above question fully, in this essay. We confess that our essay does not benefit from from any experience gained in formal teaching, nor does it derive inspiration from well- known principles of pedagogy or psychology. It emerges partly from the study of a branch of computer science, called Artificial Intelligence (a clever way to make up for our natural ignorance).

Specialists of this area of computer science have invented various concepts like knowledge base, knowledge representation, knowledge acquisition, knowledge transfer. Yet nobody knows what knowledge really is. By extension, it is not clear how we manage to transfer our knowledge from the teacher to the taught.

Primarily, teaching is influenced by an important property of the subject being taught :

- subjects where a physical support is possible or available
- subjects which involve mainly mental objects

For the first family, we cite subjects such as, say, cookery, martial arts or driving . In all these cases, the student can see and feel the subject being taught. On the other hand, there are situations which require the student to visualise the subject on a mental screen, with little or no physical support. Subjects like teaching languages, history, computer programming, or, of course, mathematics fall into this category. In the absence of demonstrable and palpable physical supports, we have to employ cleverly chosen artifacts. Our present expose is more relevant to this latter class of subjects.

"Teaching" need not necessarily be limited to classroom experiences, but can also be used in a broader sense, where knowledge from one person gets transferred to another. If you have ever tried to describe to new comer to your city, the route to reach his destination, you have gone through a rather unconventional example of teaching or knowledge transfer (of a non-palpable subject). You know your city, and have an image of the route in your mind. You want the other person to get this image (teaching/learning phase) and subsequently to find his way. How do we ensure that this goal has been met successfully ?

Our essay uses the masculine form for the teacher and for the taught. Of course, without loss of generality, we can imagine the teacher or the taught (or both) to belong to the feminine gender. In fact, computer scientists use knowledge transfer to include inanimate objects like computers, in either of these roles (as the teacher, or as the taught). With these words of caution, let us look at a few stumbling blocks, I have encountered in my long experience as a student, and my short experience as a teacher.

1.1 Abstraction

Abstraction is a very powerful tool for teaching, and for learning. Abstraction permits the teacher and the student to generalise the principles being taught and apply them to different contexts. In knowledge transfer activity, abstraction acts as the antonym of "distraction" . It helps the teacher and the taught to stay in a well defined albeit closed world, without the distracting influence of casespecific details. The common place natural numbers is an

excellent example. They allow you to count anything, such as fruits, books, money ... without having to bother whether the object being counted, is for eating, reading or spending ! And then, we build a whole lot of techniques (and more abstractions) based on this abstraction. However, abstraction is so commonplace that we hardly recognise the devil behind it. when a person says that he has five fingers in his hand, he has employed the most common form of abstraction. Ask him to define exactly what "five" means, or better still, ask him to show you just the number "five" (not his fingers), and you will see the impact of this abstraction. It is very hard to imagine how this world would look like, without the abstraction called "numbers". But, as this example clearly indicates, abstraction all alone has little value or meaning.

Abstraction can come in many forms. A geologist uses a contour map to describe the terrain he is working on. How many of us understand a contour map, or for that matter, even a simple map? Mathematicians go many steps ahead, from harmless looking numbers to frightening formulae and symbols. Chemists have their language of chemical symbols. For the intiated, abstraction provides a beautiful way of exploring and reasononing about the unknown. For the learner, sometimes this very tool can be suicidal. In fact, like in the case of numbers, we often forget that we are using an abstract concept, and we tend to frustrate an otherwise willing student. In the hands of an immature teacher, abstraction can only cause more harm than good.

1.2 Analogy

Analogy is the twin brother of abstraction, and the biggest evil in knowledge transfer exercises. An analogy would help at this point !

Seven blind men once went to "see" an elephant. All of us know the rest of this story. Analogy is good for teaching, the trouble is, the teacher never knows if the student has indeed taken the right view of the analogy. And, as the teacher continues to build up on the analogy he has introduced, the student continues to get entangled with his own interpretation of the analogy. Analogy helps, but like its twin brother, it can also be counter-productive, if used recklessly. Reducing dependence on analogy will most naturally lead to more of abstraction till we over do abstraction. A good teacher, like a good cook, must know the right recipe, to suite the needs of his clients. Analogy very quickly turns into metaphoric usage and sooner or later one tends to forget the origin of the metaphor being employed. For instance "solving" a differential equation is not exactly the same as "solving" a cross-word puzzle or "solving" a mysterious crime. Yet, the metaphor is used with great im-

punity. Any one who does not understand the term, is considered a hopeless failure. The real fault, is on the part of the teacher, who fails to recognize this metaphor and forgets to warn his students. This is but a very simple but not uncommon example of the pitfalls of analogy as a teaching aid.

The wise teacher should not only explain the "content" but also discuss the "context" as part of his teaching responsibility. This aspect is often neglected by many teachers.

1.3 Meta knowldge

Meta-knowledge, concerns things we "know" beyond the specific subject we are talking about.

The following is a true story : I once sent a young boy, with a 10 Rupee note, to buy a loaf of bread (those were good old days). The boy came back, with the bread of course, and a 2 Rupee note (they were really good old days). So far so good. But, when I ask the boy, how much did the bread cost, there was a baffling silence. The boy, an above average and an intelligent one at that, didn't know the answer to this ridiculously simple question. His parents were obviously furious. All of us adults of course "know" the answer, and expect the boy also to be able to solve this simple (arithmetic) problem. But, do we realize that there is no rule of arithmetic which explicitly mentions this fact of life ? And, do we admit that the question about the price of the bread was not an arithmetic question ?

This is what I call as meta knowledge. Although the boy knew the rules of addition and substraction, he lacked the meta knowledge, which made him blink. And, he was right in doing so . Imagine programming the best computers of today with exactly the same rules of arithmetic taught to a 7 year old, and you will have the same experience I had with this little boy. Why are we not furious when the computer blinks ? when the teacher does not recognise the dividing line between knowledge and meta knowledge, he is creating one more barrier in the student's path to learning.

1.4 Attitude

Attitude, like habits, die hard. Some of have the knack of making miserable the life of others. Maths teachers are no exception. See [1] and [2] for some convincing examples of this perverted behaviour.

2 Wrapping up

To conclude, we can draw some comfort from the fact that the difficulties faced by mathematics teachers are also common to other subjects, since formal classroom teaching is as perilous as other forms of knowledge transfer as well. In this essay, we have seen only 4 ingredients which can make or mar any teaching experience. You can avoid a lot trouble for yourself and for your students if you mix these 4 ingredients, in the right proportion, and serve the cocktail with extreme care. And, remember that like all cocktails, the same dosage does not suit everyone equally well. Just overdo any one of the components, and you will become reputed, for the havoc you create.

Let us end this essay with an equally enigmatic question: is it more difficult to teach (maths) or, to learn (maths) ?

As always, comments and remarks are always welcome, as long as they are constructive. Send your comments to the author, at : drpartha@gmail.com

3 Acknowledgements

I submit this essay as a grateful tribute to all my teachers, who have had to manage, not only with such imperfect tools, but also with a far-from-perfect student. I sincerely thank the United Nations University, International Institute for Software Technology(UNU/IIST), at Macau, for providing the facilities for preparing this article (original version).

References

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