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Technical Report

Graph colouring for resource scheduling problems – warning

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Graph colouring for a class of resource scheduling problems - a tutorial

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Abstract

This article[2] presents the of graph colouring for, resource sharing which involves conflict. A conflict is defined as a situation which necesssitates mutual exclusion. Such problems occur commonly in our daily life. Graph colouring helps the user to approach the problem in a systematic and mathematically sound fashion. A simple example is used for illustrating the approach.

1 What they don't tell you about graph colouring

Graph colouring started out from an innocent observation in the creation of multi-coloured atlases (maps). It turns out that graph colouring has many interesting and challenging aspects, and applications in other domains too. In two earlier reports [5, 6], we discussed examples of the usage of graph colouring to resource scheduling. In this report, we discuss likely problems, and the precautions we should observe with this approach.

The *contention matrix* [6] of the problem is given below.

[1] You can obtain the LaTeX source of this report, from the author. You can also download this rendered version (pdf) from the web.

	A	B	C	D	E	F	G
07:00	*			*	*		
08:00	*	*	*				
09:00	*		*			*	*
10:00	*		*			*	*
11:00	*					*	*
12:00	*				*		

1.1 Role of intuition

In the above example, we can see that the maximum number of children which can be present at any time is 4 (9:00 slot and 10:00). In each of these slots, the sets of students is different. It may seem logical to assume that 4 lockers are enough, to accommodate all students, without any conflict. We get the same conclusion when we use graph colouring [6]. Why would any one use graph clouring when the problem is solved so easily ?

Graph colouring tells us the minimum number of shared resources needed for avoiding conflict (in this case 4). By intuition, we come to the same conclusion. In addition, graph colouring also tells us how the resources are shared with 1,2,,3 or 4 lockers.

Thus the solution is that : we need four lockers:

1. Child C and child E (vertices 3 and 5) share a locker (blue colour) since they are never at school at the same time,

2. another locker (red) is shared by the children B, D, G.

3. A gets one (green)

4. F gets one (yellow)

1.2 Scalability

Instead of just 7 students competing in 6 time slots, what would happen if we had 200 students in 85 time slots ? In terms of computational complexity graph colouring is actually NP-hard. There is no known algorithm for optimal graph colouring (for a general graph) which isn't exponential; and that further, if there were a non-exponential algorithm for it, there would be a non-exponential solution for all NP-complete problems, but a non-exponential solution to another NP-complete problem wouldn't necessarily produce a non-exponential time solution for graph colouring!

In fact, the only general solution to finding an optimal graph colouring is exhaustive search: start with one node, give it a colour, assign non-conflicting colours to its neighbors, and so on. Try it with two colours, if you get no result, then try with three, and so on. There are a lot of fancy algorithms that try to improve on that - both by reducing the search space, and by using heuristics, and by trying a parallel approach. With a combination of those techniques, we can get colourings to be quite efficient in specific cases - but if always want the optimal colouring of any graph, then there's no way (or at least, no way that anyone knows about) to always get the optimal result quickly. The search becomes prohibitively expensive as the graph size increases.

In fact, this observation reinforces the fact that resource scheduling is itself a computationally difficult problem. At best, graph colouring gives us an approach to expressing resource scheduling problems in an elegant mathematical framework.

1.3 Variants of resource scheduling

The kind of problems illustrated in [5, 6] are the simplest of resource scheduling problems.

Resource scheduling can be a combination of many scenarios :

1. Resource scheduling may involve capacity bounded scheduling, where there is an upperbound on the number agents which share a resource.
2. There may be temporal constraints e.g. the minimum duration, or maximum duration for which a resource can be held by an agent.
3. There may be sequencing constraints which impose which agent should come or not come before (or after) which other agents. This class of scheduling is encountered in the control of sequential and batch sequential processes.

A simple, graph colouring approach may not be sufficient for handling resource scheduling problems of all classes.

2 Concluding remarks

The author of this paper [2] teaches discrete mathematics, to undergrad students of Computer Science, in India. He also runs a specialised enterprise which uses LaTeX and FOSS tools extensively (Algologic Research and Solutions).

You can get the LaTeX source of this text, as well as the rendered version (pdf file), by sending a request by email, to the author, at: drpartha@gmail.com. This paper, and a whole lot of similar tutorial material, is available on the web [3].

Please send your comments, remarks, and suggestions to the author :
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References

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http://www.imada.sdu.dk/ btoft/Graphcol/overview.html

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[4] A first course in graph theory and combinatorics, S M Cioba, M Ram Murty, Pub.: Hindustan Book Agency, Gurgaon, India

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