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Technical Report

Exercises in graph colouring for resource scheduling problems
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Abstract

This article presents two problems in resource scheduling. It shows how to approach these problems using graph colouring.

1 Problem statement

The following two problems are borrowed from :
http://oneweb.utc.edu/ Christopher-Mawata/petersen/lesson8.htm.
We reproduce the problems (and solutions) with permission from the orignal author Prof. Christopher Mawata (svf234@mocs.utc.edu), Department of Mathematics, University of Tennessee at Chattanooga, USA.
Details regarding the approach used are given in [1]

1.1 Problem #1

Problem 1 Suppose you want to schedule final exams and, being very considerate, you want to avoid having a student do more than one exam a day. We shall call the courses 1,2,3,4,5,6,7. In the table below a star in entry ij means that course i and j have at least one student in common so you can't have them on the same day. What is the least number of days you need to schedule all the exams?
Show how you would schedule the exams.

	1	2	3	4	5	6	7
1		*	*	*		*	*
2	*		*				*
3	*	*		*			
4	*		*		*	*	
5				*		*	
6	*			*	*		*
7	*	*				*	

The above matrix (table) is the *conflict matrix* [1] of the problem. A * in position i j (row, column) indicates that there is atleast one student who should appear in exams for subject i and subject j. Hence exams for i and j should not be held on the same day.

1.2 Problem #2

Problem 2 Suppose you run a day care for an office building and there are seven children A,B,C,D,E,F,G. You need to assign a locker where each child's parent can put the child's food. The children come and leave so they are not all there at the same time. You have 1 hour time slots starting 7:00 a.m. to 12:00 noon. A star in the table means a child is present at that time. What is the minimum number of lockers necessary? Show how you would assign the lockers.

The *contention matrix* [1] of the problem is given below.

	A	B	C	D	E	F	G
07:00	*			*	*		
08:00	*	*	*				
09:00	*		*			*	*
10:00	*		*			*	*
11:00	*					*	*
12:00	*				*		

2 Solutions

We will exploit the principle of graph colouring [1], for solving the above problems. Our starting point will be the conflict matrices [1] given above, of the two problems. From the conflict matrix, we prepare a *conflict graph*. An

arc between vertex i and j indicates a conflict between i and j, and corresponds to a * at the i j position in the conflict matrix.

2.1 Solution Problem #1

The chromatic number is 4 and a possible schedule is
Day Exam 1 1, 5 2 2, 4 3 3, 6 4 7
Note: That this is not the only four colouring possible – can you find other solutions?

2.2 Solution – Problem #2

	A	B	C	D	E	F	G
07:00	*			*	*		
08:00	*	*	*				
09:00	*		*			*	*
10:00	*		*			*	*
11:00	*					*	*
12:00	*				*		

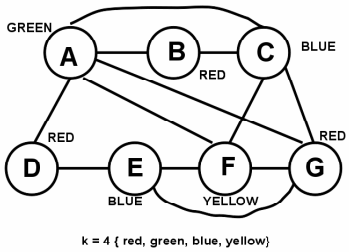
This is the *contention matrix* of the problem. It indicates the time slots at which a given child comes to the day care centre. A star in the table means a child is present at that time. He/she will need a locker, all to himself/herself.

We create a *conflict matrix* from the above *contention matrix*. A * in position ij implies that there is at least one situation when child i is in conflict with child j (for a locker), <u>at least once</u> (e.g. A is in conflict with C at the 8:00 slot, 9:00 slot and 10:00 slot):

	A	B	C	D	E	F	G
A		*	*	*		*	*
B	*		*				
C	*					*	*
D	*				*		
E	*					*	*
F	*		*				*
G	*		*			*	

It is now an easy step to transform the above *conflict matrix* into a *conflict graph*. We assign a "color" to each vertex. The two terminal vertices of each edge are in conflict (at least once), so they need two different colours.

We end up with the following *vertex coloured* graph ::



The number of colours used (aka chromatic number of the graph) indicates the number of lockers needed to avoid conflict.

Thus the solution is that : we need four lockers:

1. Child C and child E (vertices 3 and 5) share a locker (blue colour) since they are never at school at the same time,
2. another locker (red) is shared by the children B, D, G.
3. A gets one (green)
4. F gets one (yellow)

References

[1] S. Parthasarathy, Graph colouring...., Technical Report, Algologic, May 2010