Taxicabs and Ramanujan

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1 Taxicab numbers

Ramanujan [1], was a brillant Indian mathematician who was born in a poor Brahmin family, and had no formal education in mathematics, and died very young [1] (1887-1920). The story of his mentor, Prof. Hardy's visit to the hospital to see ailing Ramanujan is very well known. It gave rise to the now famous taxicab number 1729.

This anecdote is stated by Hardy as " I remember once going to see him when he was ill at Putney. I had ridden in taxicab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen".

"No," he replied in less than a wink, "it is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways." Such was the genius of Ramanujan. He was also responsible for many other remarkable contributions to mathematics [5]

The two different ways to express 1729 are:

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

It is said that the number 1729, now nicknamed as "taxicab number" [2], was also found in one of Ramanujan's notebooks dated years before the incident. In fact, we can now progressively discover many more such taxicab

[1] Srinivasa Ramanujan (22 December 1887 - 26 April 1920)

numbers by using a simple computer program [3] . It is surprising to recall that Ramanujan had no access to computers or software, when he discovered taxicab numbers. In his Lost Notebook (8.5 of Andrews and Berndt, 2013), Ramanujan presented a stunning method for generating an infinite family of solutions to the above equation [6] . His method involved the expansion of three rational functions at zero and at infinity. The integer 1729 emerges from this process.

The first few of these interesting numbers, also called as " Hardy Ramanujan numbers", are :

$$\begin{aligned} 1729 &= 10^3 + 9^3 \\ &= 12^3 + 1^3 \qquad (1) \\ 4104 &= 15^3 + 9^3 \\ &= 16^3 + 2^3 \qquad (2) \\ 13832 &= 20^3 + 18^3 \\ &= 24^3 + 2^3 \qquad (3) \\ &\vdots \qquad (4) \end{aligned}$$

An interesting point to note is that although taxicab numbers may be represented by the sum of the cubes of two numbers (in more than one way), taxicab numbers are themselves not perfect cubes (according to Fermat's Last Theorem [4]).

2 Concluding remarks

As always, your ideas, remarks and constructive suggestions are always welcome. Send them to drpartha@gmail.com.

References

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