

---

MATCHA – a simple CAPTCHA-like tool, based on  
mathematics

---

S. Parthasarathy  
drpartha@gmail.com

Our Ref.: MATCHA.tex  
Ver. : 20220419c

## Table of contents

### Contents

|          |  |           |
|----------|--|-----------|
| <b>1</b> | <b>Preamble</b>                          | <b>2</b>  |
| <b>2</b> | <b>Design principles of MATCHA</b>       | <b>5</b>  |
| <b>3</b> | <b>MATCHA Examples</b>                   | <b>6</b>  |
| 3.1      | Example – 1 . . . . .                    | 6         |
| 3.2      | Example – 2 . . . . .                    | 7         |
| 3.3      | Example – 3 . . . . .                    | 8         |
| 3.4      | More options . . . . .                   | 8         |
| 3.4.1    | Extended arithmetic operations . . . . . | 8         |
| 3.4.2    | Precedence rules . . . . .               | 9         |
| 3.4.3    | Cascaded operations . . . . .            | 9         |
| <b>4</b> | <b>Wrapup</b>                            | <b>9</b>  |
| <b>5</b> | <b>About the author</b>                  | <b>11</b> |
|          | <b>Bibliography</b>                      | <b>11</b> |

# MATCHA – a simple CAPTCHA-like tool, based on mathematics

S. Parthasarathy  
drpartha@gmail.com

## Abstract

CAPTCHA is a popular tool for preventing various kinds of fraud on the w-w-web. It is usually built into interactive forms used in web pages. We describe below, a simple and modified implementation of MATCHA. Here, we use mathematics, to raise the security of CAPTCHA by one level higher up.

## 1 Preamble

A CAPTCHA [1] is a backronym derived from: *Automated Public Turing test to tell Computers and Humans Apart*. CAPTCHA has become a generic name given to tests used for discouraging various forms of fraud on the Internet. These tools try to distinguish between automatically generated inputs and human generated inputs. Because the test is administered by a computer, in contrast to the standard Turing test that is administered by a human, a CAPTCHA is sometimes described as a reverse Turing test.

CAPTCHA (Completely Automated Public Turing test to tell Computers and Humans Apart) is a type of security measure known as challenge-response authentication. By contrast, MATCHA is Maths Assisted Turing test to tell Computers and Humans Apart.

A CAPTCHA test involves creating an activity which only humans can perform. The most common activity involves recognising a visual cue. This is followed by a challenge involving the visual cue provided. Some CAPTCHAs have been created which involve recognising an audio signal or tune (e.g. a song clipping or a spoken text). The CAPTCHA test succeeds if the user can solve the challenge correctly, based on the visual cue. Thus, CAPTCHA serves as a two-factor authentication scheme. It uses seamlessly, both the strategies:

1. What you have (the visual cue)

2. What you know (the knowhow to interpret and solve the corresponding challenge)

In the first generation of CAPTCHAs, a CAPTCHA image (of a text string) shows a random string which the user has to recognise and retype, to submit a form. This is a simple problem (seeing) for humans, but a very hard problem for computers which have to use image recognition of a displayed string which is alienated (distorted and polluted). However, such simple CAPTCHAs can be overcome by ingenious techniques, particularly Optical Character Recognition (OCR). Moreover, often CAPTCHA tests usually involve the same operation as challenge (typing-in the displayed CAPTCHA string). Subsequent improvements to CAPTCHA involve more sophisticated challenges. Some examples would be :

- Identify pictures specific objects e.g. pictures containing furniture in a mosaic of pictures.
- Counting a specific object in a picture e.g. counting the number of children in a group photo
- Naming a city (or country) based on the picture of a landmark located there e.g. naming the city where Tajmahal (shown as an image) is located
- Recognising the language from a given written script/text
- Interpret and answer a textual question (involves natural language processing) – what day comes after Tuesday ? What animal has grey stripes on a white background ?

An interesting extension to the above idea involves challenges based on mathematical texts and a basic knowledge of mathematics.

Using maths based MATCHA is not a new idea. One popular method involves generating two random numbers and challenging the user to perform an arithmetic operation (add, subtract, multiply, divide) between them. However, this method is limited in scope, since it is limited to the four fundamental arithmetic operators. Moreover, the operands should be limited to small values (usually two-digit numbers), to make it easily solvable by humans. This paper proposes an extension to this idea.

In MATCHA, we exploit the fact that image recognition is a faculty unique to humans. In addition, only humans can understand a written challenge

and choose the correct action or principle to apply in a given situation. In MATCHA, we improve upon this fundamental difference between humans and machines by exploiting a faculty which only humans have.

MATCHA is a good example of the use of mathematical knowledge in an unusual situation. The challenge involves :

1. Recognising the meaning of a written textual challenge. Applying the appropriate principles, to answer the challenge
2. Recognising and interpreting (according to the challenge given earlier) a distorted/polluted image

The two-factor authentication mentioned earlier is rendered more difficult by MATCHA.

- Mechanical image recognition of mathematical texts poses many major challenges:
  - Mathematics follows its own typographic conventions and rules
  - Mathematics uses several abstract symbols, abbreviations and acronyms
  - Mathematical texts use letters from other languages (esp. Greek)
  - Mathematical texts may contain embedded conventional texts
  - Punctuation marks (e.g. ! \* . , / | > <) in mathematical texts have special meaning.
  - The text displayed may be geometrically distorted and interspersed with visual pollutants.
- The interpretation rules (challenge) could belong to any of the various branches of mathematics (e.g. arithmetic, algebra, geometry, trigonometry, calculus, statistics ..... ). It would be very difficult for a machine to recognise and choose the right branch of mathematics and the right rule inside that branch.

In addition to its use for authentication, MATCHAs can also be used as a pictorial quiz or as a teaching aid for mathematics.

## 2 Design principles of MATCHA

MATCHA is a particular implementation of CAPTCHA. FOSS tools are eminently suitable for creating MATCHA.

A typical MATCHA test follows the steps below:

1. The computer presents a randomly selected visual cue and a challenge
2. The human types in the response to the challenge, based on the visual cue
3. If the response is right, the human is allowed to continue
4. If the response does not match the expected value, the human is blocked or rejected

Each MATCHA test is a triplet : <a visual cue >, <a challenge >, <a correct solution to the challenge >. The entire MATCHA can therefore be stored as a 2 dimensioned array, each row of which, corresponds to a MATCHA test. The array may contain any number of rows (MATCHA tests). The tests are chosen randomly from this array. This approach is however considered to be insecure, since the array is visible to everybody.

They can also be stored as a database. Each record of the database will define a MATCHA test and will consist of the above three fields. These records can be integrated into HTML forms using PHP and other server-side scripting tools. Access to the database can be restricted by using password or access keys. This makes the CAPTMA more secure than the array based approach.

However, the most secure option would be to generate the MATCHA images and challenges, dynamically. This does not involve storing anything, but is more cumbersome to program. Creating a visual image dynamically, and adding visual distortions to it is not very easy.

Using mathematics for the visual cues is an extremely rich choice, since it can be visualised by the creator himself. It is easy to create the visual cues for each MATCHA test, using tools like GIMP, LaTeX, MathJax etc. The MATCHA developer can generate a huge number of visual cues, and the corresponding challenges, using his/her own creativity, imagination, and innovation. The difficulty level of the corresponding challenges can be chosen

by the MATCHA designer himself/herself. However, it should be remembered that MATCHA is not a test of the mathematical skills or knowledge of a person.

A few simple precautions will make it difficult for automatically/mechanically solving the MATCHA test:

- The visual cue should be simple and compact. It should be possible to observe the entire cue in a single eye-shot. Visual distortions and graphical pollutants should make it difficult for automatic image recognition, but should not render the image illegible to a human.
- The challenge itself should be worded briefly, in unambiguous terms and without using any specialised/unusual terminology. For every MATCHA test, the challenge should be worded differently.
- Care should be taken to see that the wording of the challenge is different for different cues.
- The solution should be unique. It should be possible to type down the response, on a conventional keyboard, or be chosen by a mouse-click.
- Part of the challenge should be displayed in the cue, graphically. In other words the challenge cannot be interpreted without first recognising the cue.
- The solution field itself is never displayed. It is used invisibly for verifying if the response entered by the user is correct or not.
- It would not be a good idea to let the solution be visible anywhere in the visual cue or challenge. For instance, a bad choice of the challenge would be:

Who is the odd man out in the sequence : 12 6 8 12 7 42 66.

In the above example, it would be easy to defeat the above challenge by a trial-and-error approach or exhaustive enumeration of all the numbers.

## 3 MATCHA Examples

### 3.1 Example – 1

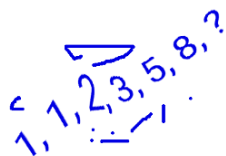


Figure 1: Example 1

The challenge for this example is : *What is the last number in this sequence ?*

In fact, it would be very difficult for a machine to locate and recognise the sequence in this graphically polluted and distorted picture. Added to all this, is the fact that understanding the text of the challenge, is not at all trivial. It would be much harder to recognise that it is part of a Fibonacci sequence, since there are several similar integer sequences [2]. By comparison, any human (with some knowledge of mathematics) can spot the answer in no time.

The user is considered to have succeeded when he/she enters the correct answer (13) in the appropriate box in the form where this MATCHA challenge is displayed.

### 3.2 Example – 2

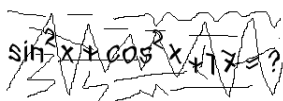
This challenge is based on a trigonometrical identity. A machine may find it difficult to recognise the distorted image of a text like this, and to infer that it is a mathematical expression. It would be much harder (for the machine) to recognise (and solve) the hidden riddle.

$$\sin^2 x + \cos^2 x + 17 = ?$$

Figure 2: Example 2

By comparison, this same challenge would be a trivial task for any mathematics-aware human being.

The above image could be rendered more difficult to interpret mechanically. An example is shown below. In this case, we use two graphic methods, super-imposed one-over-the-other, to create a visually polluted the text:



- Distort the basic text
- Add background noise

Care should be taken so that the polluted image does not become illegible to a human.

### 3.3 Example – 3

A commonly used challenge involves two numbers and an arithmetic operation over these two numbers.

Instead of displaying the above challenge, in a text form as :



$$1472 - 907 = ?$$

we will display it graphically after some graphical distortions, like in this image on the left. Conventional text display can be recognised by OCR tools, whereas the graphical display is much harder to recognise and interpret (for a machine). Notice that the arithmetic operation to be done is spelt out and graphically displayed instead of as a simple textual character ( - ).

However, challenges which involve the four basic arithmetic operations, are limited in scope. They can be broken by an exhaustive search over all the four operations. We explain below, a strategy to overcome this limitation.

### 3.4 More options

We can extend the above idea, to make it harder to mechanically interpret MATCHA images. We can try any combination of the following strategies.

#### 3.4.1 Extended arithmetic operations



We can try going beyond the four basic arithmetic operators. We can use many (simple) binary arithmetic operations, to discourage automatic/mechanical interpretation of the MATCHA tests. The possibilities are endless.

A few possible options are :

- $\text{gcd}(a,b)$
- $\text{lcm}(p,q)$
- $x \bmod y$
- $\text{average}(x,y)$

The operands must be small (one or two digits) so that the user can solve the operation with just a simple eye-shot.



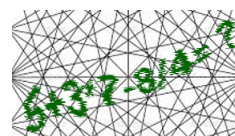
### 3.4.2 Precedence rules

We can extend this to expressions which involve more complex rules. Here is a simple arithmetic expression which any schoolboy can solve:

$$5 + 3 * 7 - 8/4 = ?$$

provided he knows the rules of arithmetic precedence.

If we present the above expression as an image (with a noisy background), we will have to first extract the above expression from the image, recognise it as an arithmetic expression, parse the expression respecting precedence rules, and arrive at the solution. This would indeed be a major effort to automate.



### 3.4.3 Cascaded operations

Arithmetic over cascaded operations and nested parantheses is generally tricky. Here is an eaxmple:

$$((-7) * (-7) + ((-7) * (7)) * 4 + 5$$

**Warning :** All the above strategies may be combined in any sequence, to achieve any level of difficulty for automatic recognition and interpretation. However, since this is not a test of the mathematical skills of the user, it should be used with care. This type of challenge may not be suitable for a layman audience.

## 4 Wrapup

This is a “proof-of-concept” article. It provides outlines of an unusual usage of mathematics, nicknamed MATCHA, to provide a simple but powerful security extension of the CAPTCHA principle using the following potent facilites:

- Mathematics offers limitless possibilities for creating MATCHA challenges.
- Many FOSS tools exist for implementing MATCHA.
- These strategies can be combined and used together.

A prototype implementation of MATCHA is under development and testing. Several improvements can be made to the prototype implementation. Currently the entire captcha tests are hard coded into a single array in the PHP code. One can transfer this information to an external database, and protect it using access keys or passwords.

If you find this article useful, please send a note to [drpartha@gmail.com](mailto:drpartha@gmail.com). Comments, suggestions and remarks are always welcome, as long as they are constructive. This article, as well as other similar articles can be downloaded from [3]

## 5 About the author



S. Parthasarathy (Dr. Partha) brings with him, a very rich experience (since 1980) in the software industry. He usually teaches discrete mathematics, to students of Computer Science, in India. He is an aggressive supporter of the Free Libre Open Source Software (FOSS) movement, and is a regular contributor to the international effort on Linux/FOSS. He also promotes the use of  $\text{\LaTeX}$  for all academic and technical publications. He was decorated by IEEE (USA) for designing excellent websites for IEEE. His

Consulting company provides assistance in web design and maintenance.

You can contact him, by e-mail, on : [drpartha@gmail.com](mailto:drpartha@gmail.com) . For more details about Dr. Partha, please visit : <http://drpartha.org.in/drpartha.htm>

## References

- [1] Wikipedia, CAPTCHA,  
<https://en.wikipedia.org/wiki/CAPTCHA>
- [2] OEIS, The On-Line Encyclopedia of Integer Sequences (OEIS)  
<https://oeis.org/>
- [3] S. Parthasarathy,  
<http://drpartha.org.in/publications/downloadables.htm>

\* \* \* \*