How Does a Mathematician's Brain Differ from Other Brains?

Ali 5 days ago · 12 min read ★

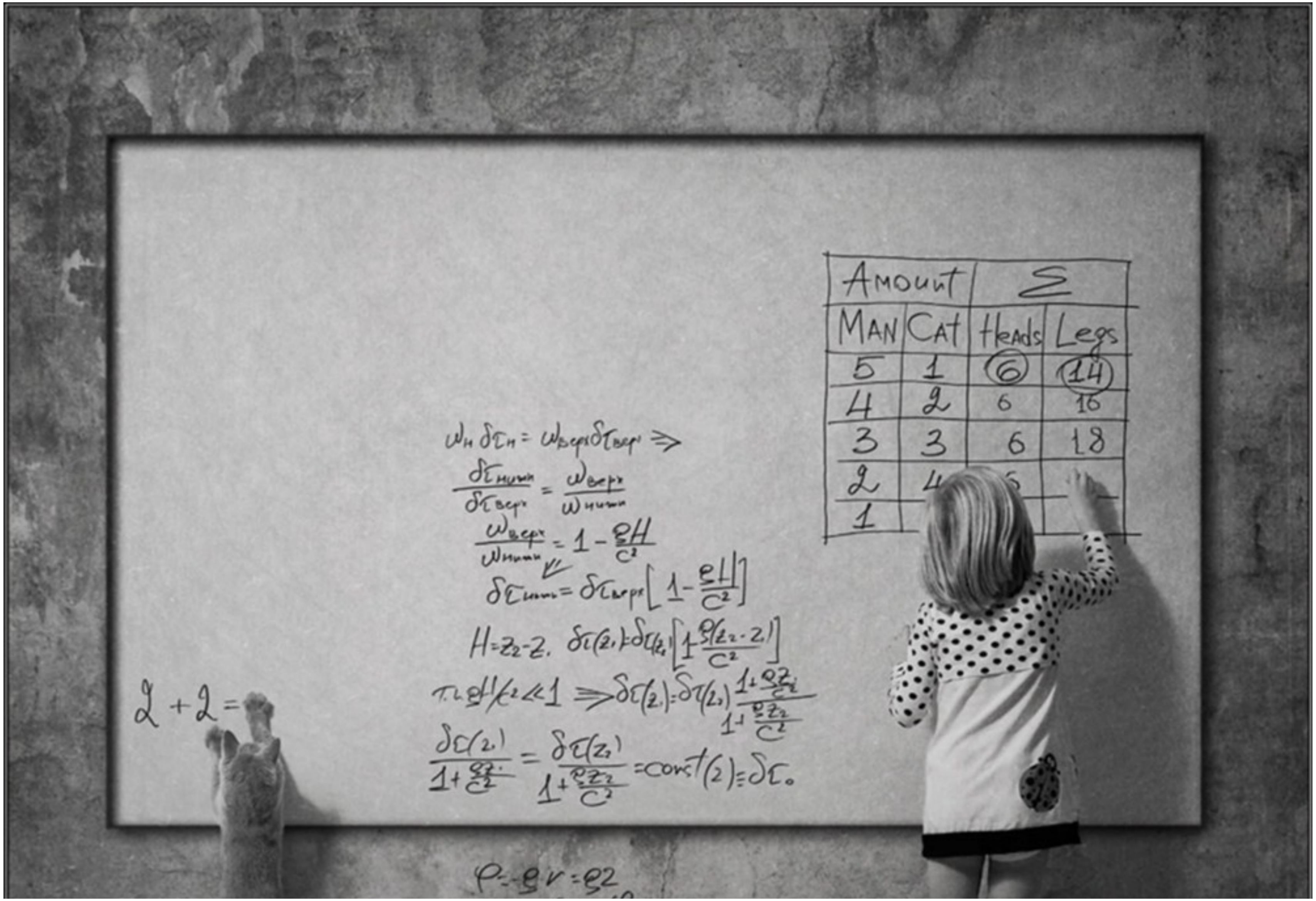

Amount
Σ
Man
Cat
Heads
Legs
5 1 6 14
4 2 6 16
3 3 6 18
2
1
2 + 2 =

Photo by Andy Prokh | Source: today.com

The question of how mathematicians think is closely related to the question **"How does a musician compose?"** Similarly, this question is asked to learn how the creative process works. Those who are interested in computer science or especially artificial intelligence can give the correct answer to this question. A real mathematician is not interested in finding the answer to this question. He's just busy doing math.

Unfortunately, there is no clear way to answer the question of how a mathematician thinks. But we can approach this question as follows; if you watched any chess tournament, the game's analysis is shared in detail at the end of the match. When you examine the analysis, you will see a breaking point in each game. Similarly, mathematicians also experience a breaking point while working on a problem before finding a solution.

Therefore, it is helpful to analyze a few mathematical proofs to answer our question and pinpoint the breaking points. For example, as we all know, Euclid’s theorem states that there are infinitely many prime numbers.

The proof of this theorem is more beautiful than the theorem itself.

So how did Euclid realize that prime numbers are infinite? Euclid’s approach to the solution is fascinating.

```
First, Euclid assumed that prime numbers are finite. He then
constructed a set where he wrote all the prime numbers and called the
elements of the set P = {p1, p2, p3,…, pr}. So, by that assumption,
any number other than these numbers should not be prime. Then Euclid
multiplied all the elements of the set P and added 1 to the product.
Then he got a new number and called this number N.
```

Besides this, Euclid has some excellent math knowledge he has gifted with, such as the **Fundamental Theorem of Arithmetic.**

```
For example, he knows that if a number is not prime, it can be broken
down into prime factors. Thus, when he tried to factorize the number
N since N isn’t a prime number, N must be divisible by at least one
prime number. However, all the primes are here, and they cannot
```

divide it because of the plus 1. For example, 2 and 3 make prime numbers, and **2 x 3 = 6**. 2 and 3 can perfectly divide the number 6 but not 7 (6+1). So **there is a contradiction** here. The number **N** is not in the set of prime numbers, but it is prime. So prime numbers must be infinite.

Proof of Euclid’s Theorem

When we look at the proof, we see that Euclid does something different from the norm. Contrary to what we might think, the idea that prime numbers are finite is not creative. Mathematical logic is already directing us to with a start a proof like that. **The truly original idea here is when Euclid threw out the number N**.

Understanding Euclid: A Simplified Approach to Mathematical Thinking

I remember that my mathematical education has started with numbers. First, my father, then later, my elementary...

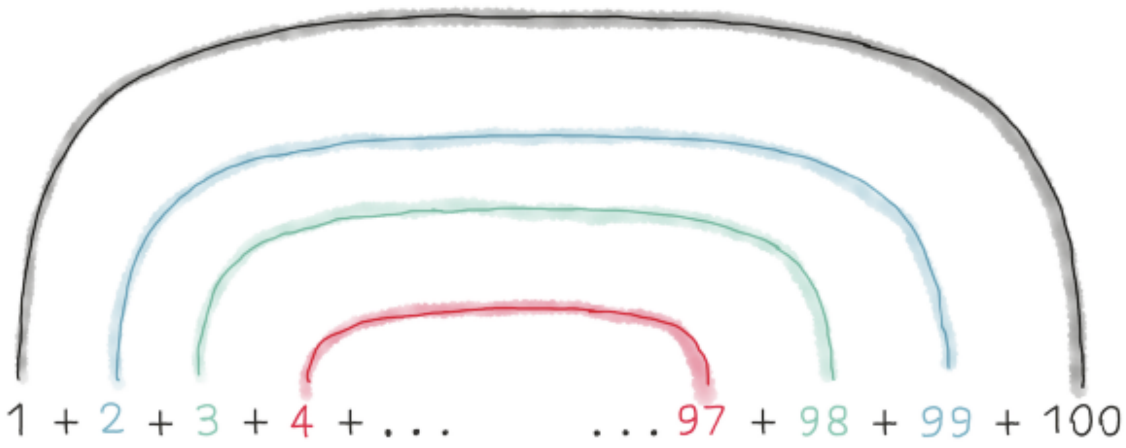
medium.com

Let's try to prove another theorem.

When the children in an elementary school were very naughty, the teacher wrote a difficult question on the chalkboard to silence all the children. The teacher asked the children to add up all the numbers from 1 to 100.

On that day, the young German boy named Gauss, who would grow up to be one of the greatest mathematicians of the future, was in that

```
class. While the teacher thought it would take a long time for the
children to solve the question, Gauss resumed talking to his friends
again in just a few minutes. When Gauss's teacher asked why he was
talking, he said he had already solved the question. That day, all
the students in that class tried to add all the numbers one by one,
but Gauss did something unusual. He saw that he would always get 101
if he added a number from the left of the sequence and a number from
the right. For example, 1+100, 2+99, 3+98,…, 50+51; it was always
101, and there were 50 of them.
```



If we look carefully at this method, we will see that this is an observation that even a small child can see. However, it is a fact that not all young children solve this question like that. **Furthermore, many intelligent people who are much older cannot see this simple solution.**

In short, we have many good examples to analyze how a mathematician thinks. Almost every mathematician has been adapting and manipulating theorems and the proofs that have been introduced to them before to find a new idea or solve a problem. But **those who are considered geniuses are doing a few extra things.**

For example, let's consider a musician. He doesn't have to be Johann Sebastian Bach or Igor Stravinsky. It is enough for him to be an ordinary musician who does his job correctly. Let's say our musician goes to the conservatory and studies for four years and completes his master's and Ph.D. Now, if we ask him to make a composition, can he do it? Of course, he can, but we can't be sure if he can compose something like Bach's. Anyone can learn some things by education or training. A student who studies in the mathematics department for four years will surely learn mathematical thinking. But it is unlikely to be something like **"give a one-hour education and expect a new theorem in return."**

Mathematical thinking education is a continuous process. Students first take abstract mathematics lessons. Then they learn how to prove that prime numbers are infinite, as in the example above. Then they realize they can also prove another theorem in the same way. For example, using the same method, you can find infinite “p” prime numbers for p ≡ 3 mod 4. In short, to be a mathematician, the main thing is adapting smaller, known proofs to larger, unknown proofs. These skills become internalized after adequate practice.

The level of education received has a significant effect on the formation of the mindset. For example, **when mathematicians and physicists come together and discuss, they can never meet at a common point.** While physicists look at a problem from one window, mathematicians view it from a different place.

```
For instance, when you mention Lie algebra to mathematicians, they
describe the bracket operation directly. However, when you say it to
physicists, they follow the path of the structure constant and do
their work accordingly. The approaches to the same problem can be
drastically different.
```

> Mathematicians generally have a particular way of thinking, and this state of thinking emerges with homework, questions asked in exams, and textbooks read. Again, this is a very long process. It takes a long

time.

So far, I have only talked about how an average mathematician thinks. Here, I find it useful to mention some critical figures when discussing the development of mathematical thinking. One of these figures is the Indian mathematician Srinivasa Ramanujan, whose life was not very long. His mathematics education was different from the education I mentioned above. Moreover, Ramanujan's college performance, other than in mathematics, was so bad that he was kicked out of the university. **The only math book Ramanujan read was an arithmetic exercise book that no mathematician would ever read.** However, Ramanujan was a mathematician who wrote huge math formulas and collected them in a notebook. One day, Ramanujan sent that notebook to the famous English mathematician Godfrey Harold Hardy, a Cambridge professor.

Ramanujan and Hardy at Cambridge | From the movie The Man Who Knew Infinity

Of course, since the hundreds of letters were coming to Hardy from everyone in those days, he didn't take the message from India very seriously at first. But then, the formula he saw on one side of the letter caught his attention because it was very similar to a question he was dealing with in those days.

> "This man is not a charlatan..."

Later, Hardy and his best friend Littlewood started to study all the formulas Ramanujan sent. They couldn't even understand some of the notes. Hardy first used the phrase **"This man is not a charlatan"** for Ramanujan. When Littlewood asked why, he said in reply, **"Because it is not even possible to make up those formulas he wrote. So those formulas must be correct."** Hardy invited Ramanujan to England immediately in the reply letter.

Hardy and Littlewood noticed something very awkward when Ramanujan arrived. **Although Ramanujan could write infinite sums as equality, he knew nothing about modern mathematics.** So they asked him to learn modern math and take analysis lessons. But then something more awkward happened. **Ramanujan could not understand the common epsilon-delta technique,** the basis of abstract thinking. Indeed, this was a very unusual situation for students studying mathematics.

Building Abstract Thinking in Math Using Epsilon-Delta

When I close my eyes and go back in time, I see a college student sitting in the back row and looking sad while the...

medium.com

Ramanujan was a genius, and we could not understand how he thought, unlike Euclid. Despite solving infinite sums, he could not understand the most basic analysis technique. Ramanujan did not have the slightest idea of complex analysis, but he could work on zeta functions. So Ramanujan had a different mindset in his mind that only he knew, and one we will never understand.

```
One day, when Hardy wondered about this situation and asked how he
```

> wrote all those formulas, **Ramanujan told him that God gave him all the formulas, and he had just written them.** To me, this was a very reasonable answer because Ramanujan was a math practitioner 24/7, and he often forgot to eat. His wife or mother reminded him that Ramanujan had to eat. The few times he did sleep, he continued doing math in his dreams.

Such things happened to me during the finals week when I was a student. Once, I was working so hard on the Calculus exam, and I continued to solve some questions in my dream at night.

In England, Ramanujan met his childhood friend Chandra Mahalanobis. Mahalanobis was the founder of a very famous institute called the Indian Statistical Institute at the time. Mahalanobis was a student and a good statistician at Cambridge. Ramanujan and Mahalanobis were 1st and 2nd in a math contest in India when they were children. Ramanujan was 2nd, and it is said that he cried for days saying that he was the greatest mathematician himself.

> One day Mahalanobis asked Ramanujan a question:

> "There are between 50 and 500 houses on the street, and all the houses are numbered 1 to n. The sum of the numbers of the houses on the left of a house we choose must be equal to the sum of the numbers of the houses on the right. How many houses like this could he find?"

After some calculation, Mahalanobis found out that the 204th house on a 288-housed-street achieved this equality and shared it with Ramanujan. As soon as Ramanujan saw the question, he said if there were no other conditions, we could solve it by taking a continuous fraction. This question will later be one of the arguments underlying Ramanujan-Rogers' identities. Returning to the topic, as soon as he saw the problem, Ramanujan realized that the solution was related to fractions. **An average mathematician, on the other hand, cannot come to this conclusion so quickly.** So the exciting example of Ramanujan is a different perspective on the question of how a mathematician thinks.

```
Ramanujan has another similar story: the taxi plate case.

When Ramanujan got very sick and was taken to the hospital, Hardy
took a taxi and visited him immediately. When the atmosphere was
silent for a long time, Hardy said that the number plate of the cab
he rode was quite ordinary; 1729. Ramanujan replied, without
hesitation, "How come it is not interesting? It is the smallest
number that can be collected as two different cubes".
```

Gauss and Ramanujan are geniuses who internalized mathematics and numbers to a large degree. The common point in these types of mathematicians is that they are not afraid of calculating the size of the calculations. But we should know that great mathematicians also think differently from those we have mentioned so far and were never accept their way of thinking. Unlike the mathematician Ramanujan, Alexander Grothendieck, who studied mainly algebraic geometry, dealt entirely with abstract mathematics. Algebraic geometry is generally concerned with roots that reset polynomials.

In algebraic geometry, the mathematician Andre Weil previously had some problems that could not be solved for a long time. Pierre Deligne, a student of Grothendieck in the 1970s, proved the Weil conjectures. Grothendieck was very angry with his student when he used a theory about Ramanujan's modular forms in his proof. Because he saw that Deligne used one of Ramanujan's theories in his proof, Grothendieck was considered a trick. **According to Grothendieck, proof must be simple and nothing extra.** That's why he never liked Deligne's proof. As can be seen in this example, **Ramanujan and Grothendieck were two mathematicians with very different mindsets.**

I want to prove another example. According to one theorem, **there are irrational a and b numbers such that a to the power of b is a rational number.** We can prove this theorem as follows.

> Let's take the number square root of two. We know that **the square root of two is irrational.** Let's investigate the square root of two to the power of the square root of two. There are two situations. It is either rational or irrational. If it is rational, we are done because we immediately pick the numbers a and b as the square root of two, and the case is closed.

```
However, if the square root of two to the power of the square root of
two is irrational, the square root of two to the power of the square
root of two to the power of the square root of two becomes the square
root of two square. That will be equal to 2. So this makes the result
rational. So we encounter a contradiction.
```

If we take this proof as an example of mathematical thinking, it will be sufficient. However, some mathematicians do not like this method of proof. Such methods have always been at the root of the conflict between David Hilbert and L. E. J. Brouwer. How a mathematician thinks is an important question, but there are also differences in the way of thinking among mathematicians. One does not like the other's way of thinking. Some mathematicians don't even like **the axiom of choice.** But we use the axiom of choice when performing function analysis. So we use it even in the most basic things.

Now, we talked about rational mathematical thought and its forms, but there is another layer to it as well. Mathematics is getting more complicated by the day. Some proofs can take hundreds of pages. For example, the classification of finite simple groups has proof that fills thousands of pages. There are even rumors that there was a mistake somewhere that says there should have been a few other groups among the sporadic. That is why **some mathematicians are currently working on whether it is possible to make computer programs that can control the proofs or, better, computer programs that can directly make the proofs.**

The same goes for music because of music and mathematics progress in parallel. Musicians are also dealing with similar things like mathematicians. For example, there are music composers for the computer. There is a specific speed limit for humans, we cannot type eight symbols simultaneously, but you can do anything on the computer. So there are people who wonder whether we can find a place for the computer in mathematical thinking. That idea raises the question:

> *Can a computer start doing mathematics on its own? Alan Turing is the first to ask this question. As a closing thought, while we examine how a mathematician thinks, we should also start thinking about what a computer can do if it starts thinking like a mathematician in the future.'*

- * Note: I get commissions for purchases made through links in this post.
- ** I wrote this article after I watch one of my favorite professors Ilhan Ikeda's lecture on Youtube. Since it was a beautiful lecture, I decided to turn my lecture notes into article.

Mathematics Philosophy Education History Teaching